

# On Reducing the Effects of Aliasing on Dynamically Estimated Radio Frequency Maps

Garrett Vanhoy\*, Haris Volos\*, Mohammed Hirzallah\*, Carlos E. Caicedo Bastidas\*\*, Tamal Bose\*

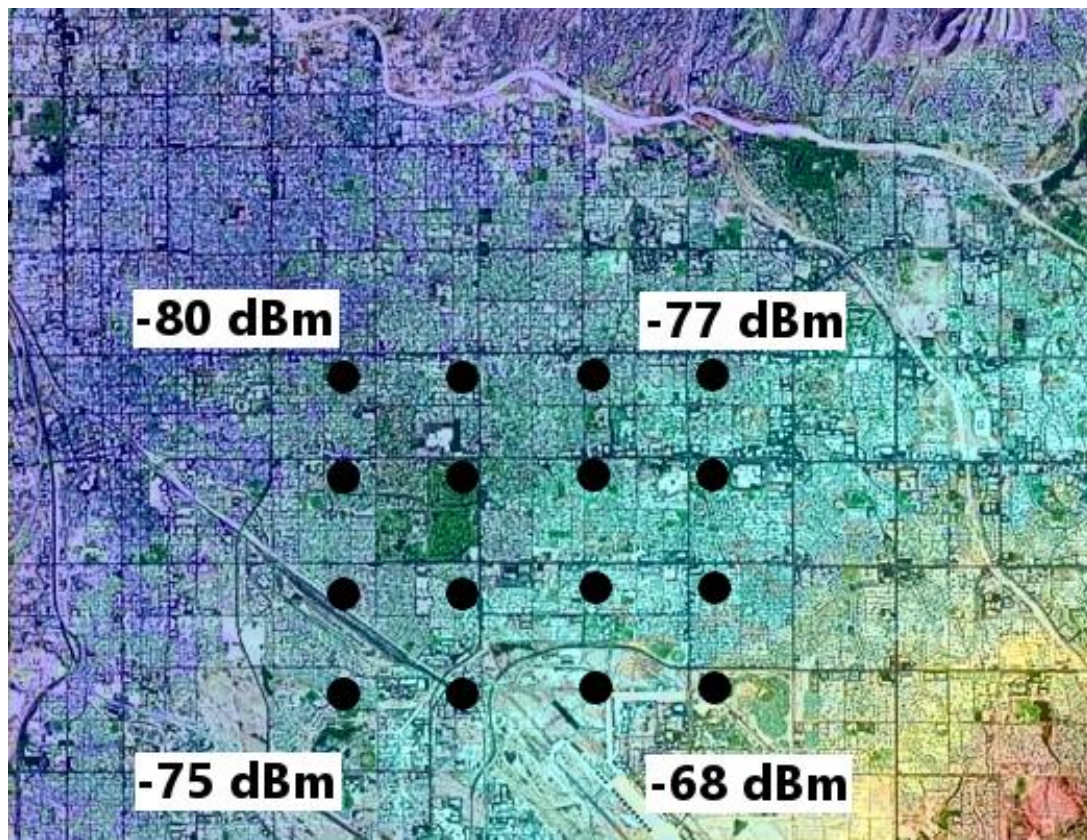
\*Dept. of Elec. and Comp. Engr.  
The University of Arizona  
Tucson, AZ 85721-0104  
{gvanhoy, hvolos, hirzallah,  
tbose}@arizona.edu

\*\*School of Information Studies  
Syracuse University  
Syracuse, NY 13244  
ccaicedo@syr.edu



# Agenda

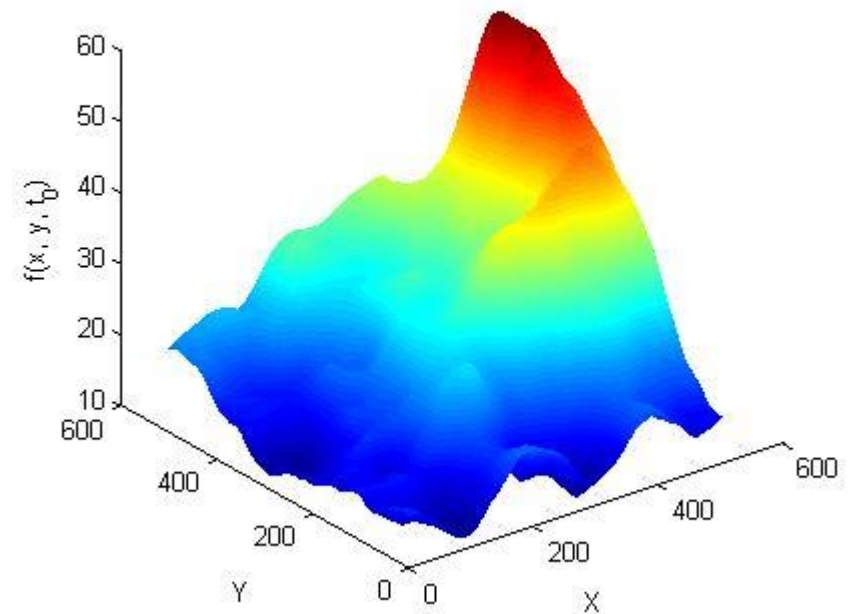
1. Dynamic RF Mapping (DRFM) Challenges
2. Aliasing in DRFM
3. Aliasing Reduction by Local Power Estimation
4. Results



## 1. Dynamic RF Mapping

# Dynamic RF Mapping

- The RF map can be described as  $f(x, y, z, t)$  with  $(x, y, z) \in \mathbb{R}^3$ 
  - This describes the *average* RF power at a point in 3-dimensional space over time within a bounded space.
  - For simplicity, we study the RF map at one elevation  $z_0$ . Thus, we have  $f(x, y, z_0, t) = f(x, y, t)$ .
- How can we accurately estimate the shape of the surface:  $\{(x, y, f(x, y, t_0)) | (x, y) \in \mathbb{R}^2\}$ ?





# Why DRFM?

- It can be helpful for many reasons:
  - Wireless network coverage enhancement / studies, interference management, spectrum policing, spectrum sharing, and others
  - A rough estimate of the RF map is still very useful for many applications
  - DRFM will be most useful to use between systems that cannot communicate

# Methods for RF Mapping

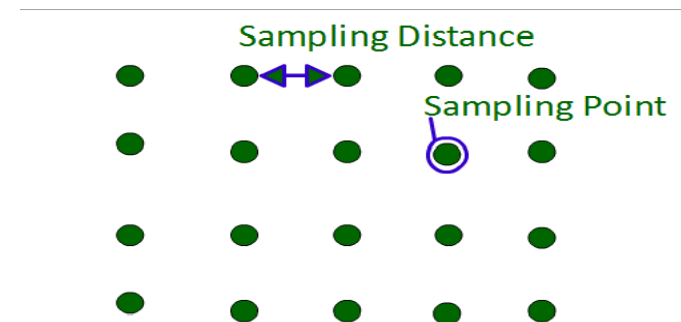
- **Use propagation models**
  - Model-fitting (transmitter location(s))
  - Ray-tracing (physical terrain)
- **Interpolation**
  - Using sample points alone to estimate the RF map (continuity, second-order stationary)
- Dynamic implies a rapidly changing RF map. This change could be due to movement emitters, changes in environment, or movement in the sensors used in RF mapping.
- For a *dynamic* environment, interpolation methods are more suitable

# Building RF Map

- Kriging (statistical approach) [3]
  - Used widely in geostatistical modeling
  - Assumes second-order stationarity, subject to the validity of the semi-variogram
- Inverse Distance Weighting [2]
  - Interpolates points as a weighted average of nearby samples
- Thin-plate splines [1]
  - Polynomial fitting of the data-points
- Discrete Cosine Transform

# Aliasing in DRFM

- There still a considerable amount of Error even though we are using good interpolation techniques. Why?
- Answer is because we are doing sampling, but we could not satisfy Nyquist
- Nyquist Rate:  $f_s \geq 2f_{\max}$
- Sampling distance, not sampling period.
- The sampling distance is related to the level of detail. High details mean smaller sampling distance.





# Previous Work

- The most successful attempt at minimizing the number of sampling points in a Rayleigh fading environment uses compressive sensing.\*
- Still requires nearly the same number of sampling points to satisfy the Nyquist condition – suggesting an RF map is a low-pass type signal.
- Is it then possible to overcome this?

\*Y. Mostofi, “Compressive Cooperative Sensing and Mapping in Mobile Networks,” *IEEE Transactions on Mobile Computing*, vol. 10, no. 12, pp. 1769–1784, Dec. 2011.



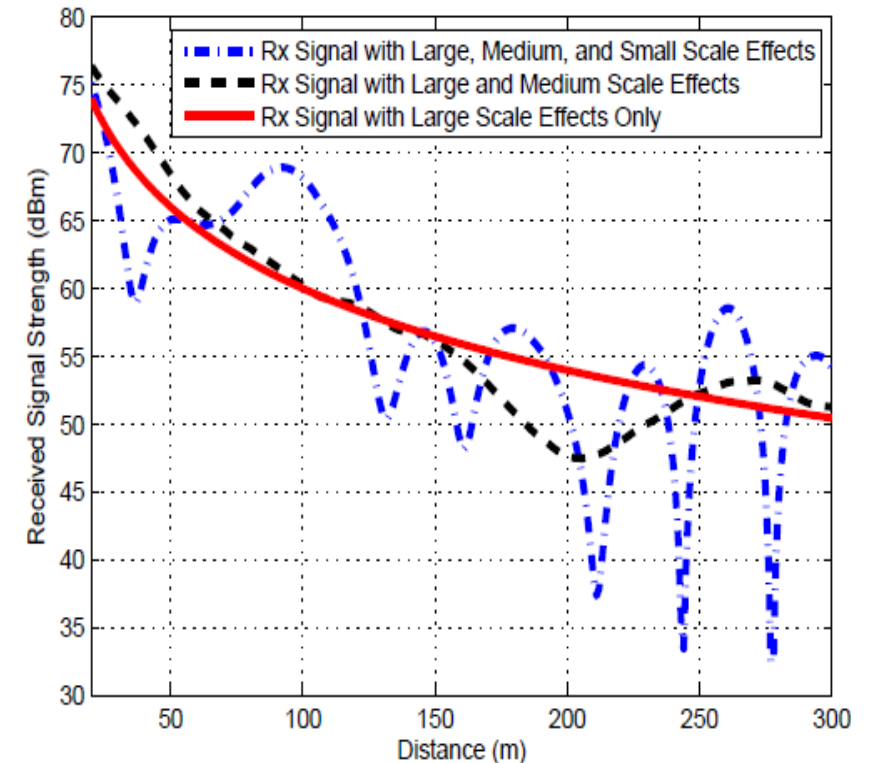
## 2. Quantifying the Aliasing Effect

# What contributes to aliasing?

- Aliasing  $\leftrightarrow$  Bandwidth
- Signal Propagation is affected by three main factors:

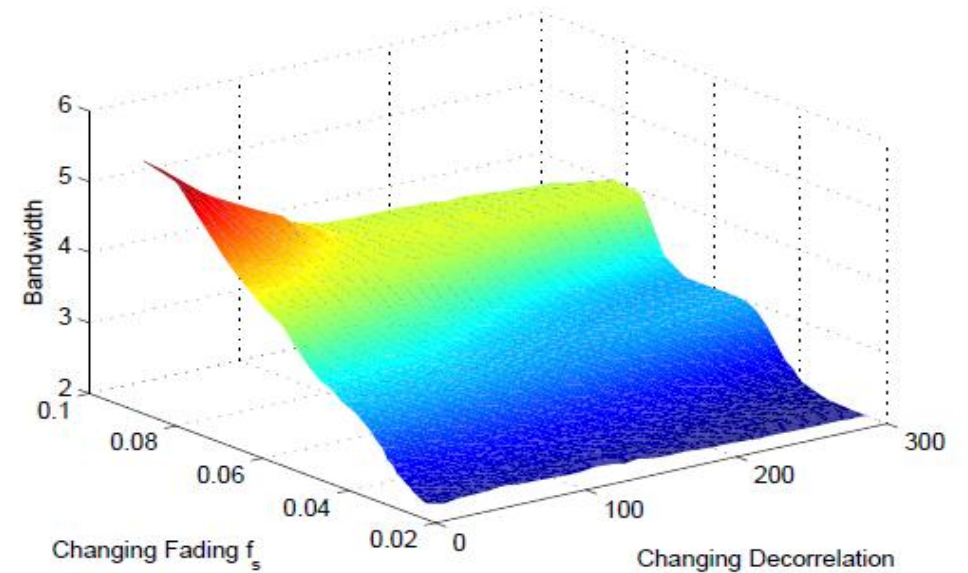


- High variations contribute to a higher bandwidth
- Bandwidth here is in the wavenumber sense, not in the Hertz sense.
  - Time  $\leftrightarrow$  Hertz  $\text{sec}^{-1}$
  - Distance  $\leftrightarrow$  Wavenumber  $\text{m}^{-1}$



# What contributes to aliasing?

- By varying shadow-fading correlation distance and small-scale fading Doppler-spread we can observe their effect on the bandwidth





# Implications

- Fading relates to bandwidth which relates to aliasing and error in map estimation
- To reduce the aliasing effect DRFM methods should focus on mitigating the effects of small scale fading
- One way this can be accomplished is by applying local power estimation averaging



### 3. Aliasing Reduction by Local Power Estimation

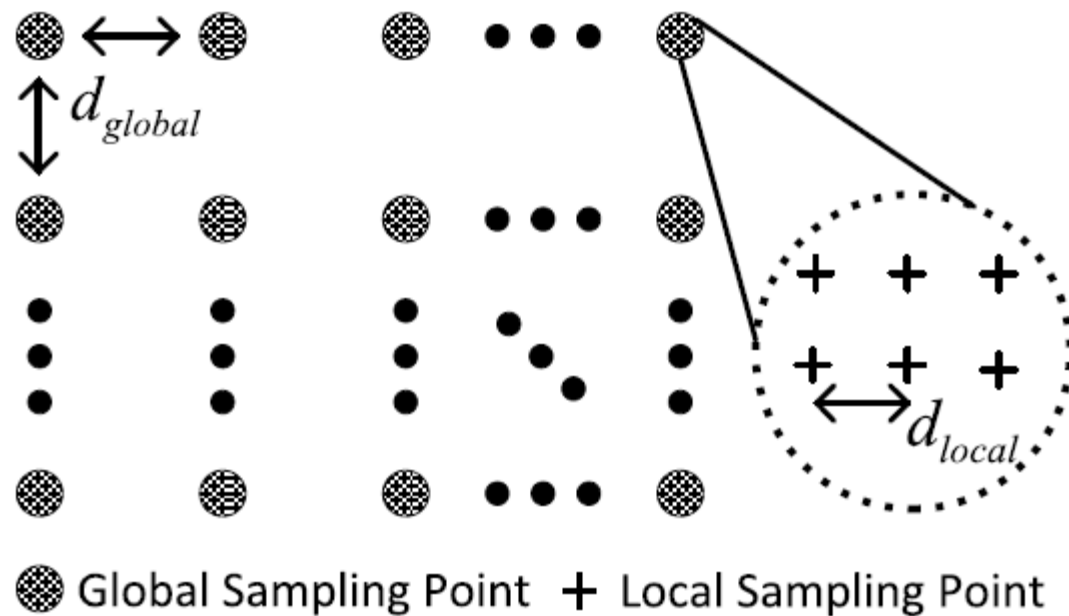


# Approach

- Do not estimate the RF map that includes fading
  - We want a good idea of what the RF map looks like in a given area
- Estimate the RF map as if fading was not there

# Local Power Estimation

$$\frac{d_{global}}{d_{local}} > 50$$





# Local Power Estimation (cont'd)

- **Estimate the local average power using the local samples**
  - Take measurements from several 'local' samples
  - Combine them through averaging or through another statistical estimator
- **Interpolate from local power estimates**
  - 'Global' samples will be interpolated using DCT to build the final DRFM.
- It is worth mentioning, that the 'local' samples will be subject to correlation



## 4. Results

# RF Map Generation

- Step 1. Two Ideal maps were generated: one with fading other without fading.
  - Path loss exponent of 2
  - Shadowing decorrelation distance of 50 meter distance and 6 dB standard deviation
  - Small-scale fading introduced by Rayleigh distribution at each point
- Step 2: Rayleigh fading at each point is averaged to generate global samples
  - Or an optimal estimator for a Rayleigh distribution is used:

$$\frac{(N - 1)! \sqrt{\pi T}}{2\Gamma(N + .5)}$$

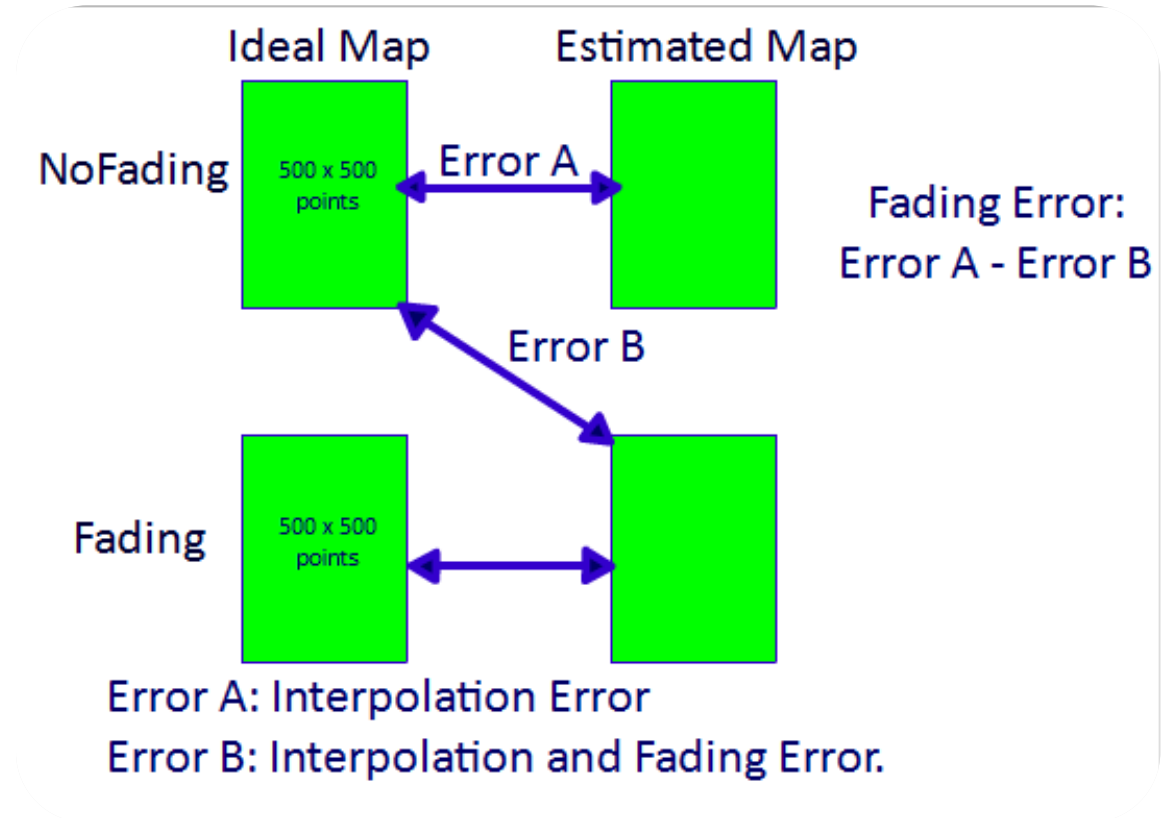


# Estimation Error

- Step 3. The two maps were resampled using an arbitrary sampling density to simulate measuring the RF map at a given point
- Step 4: These measured points were interpolated using DCT to the size of the original RF map for a point-by-point difference

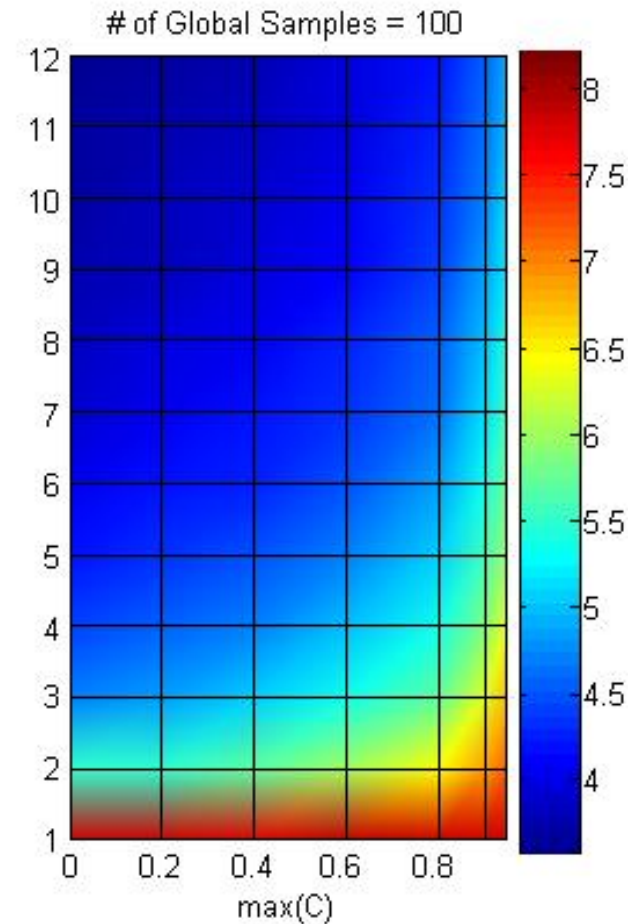
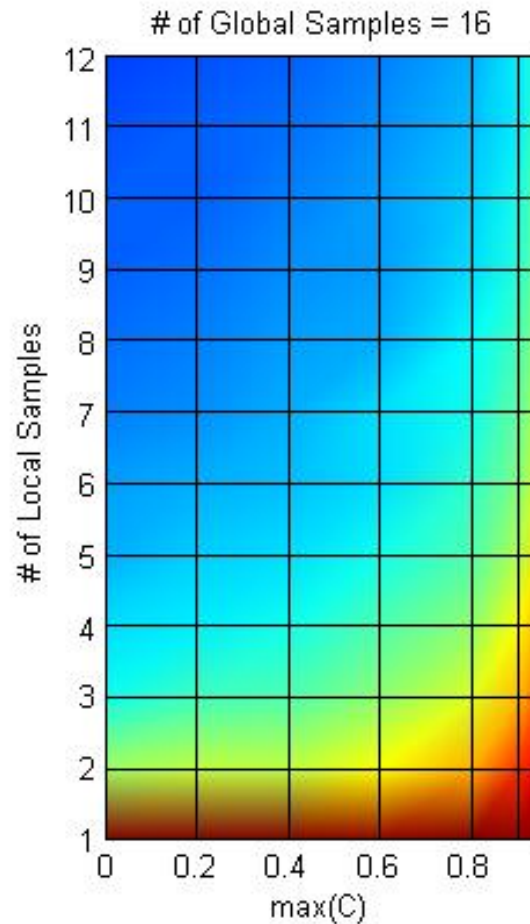
# Evaluation

- **Two Metrics:**
  - Ability of the proposed method to approximate the ideal map without fading (**Error B**)
  - Additional estimation error as a result of fading (**Error A – Error B**)
- **Trade-off Analysis**
  - View the effect of correlation and the number of antennas on each metric



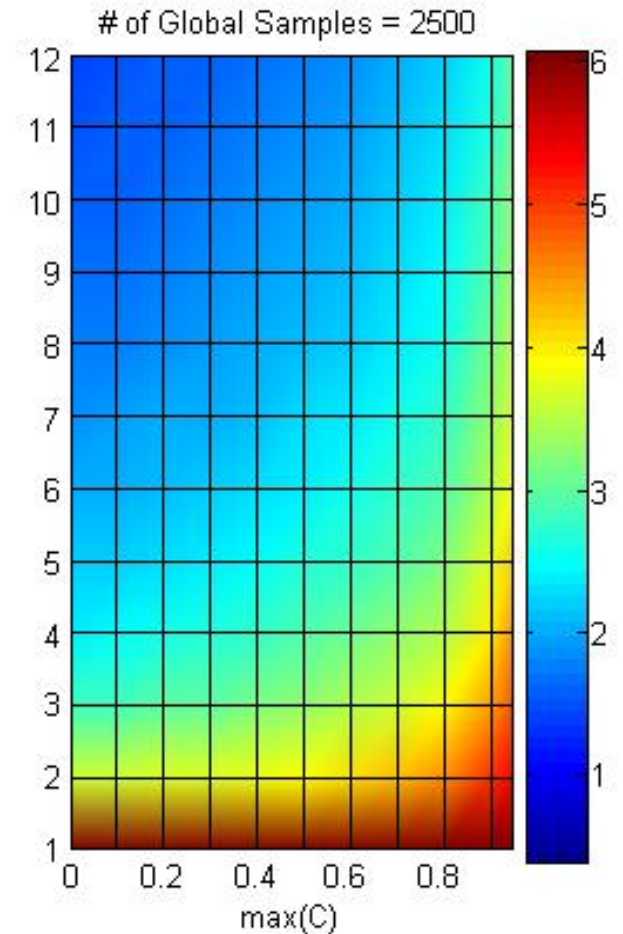
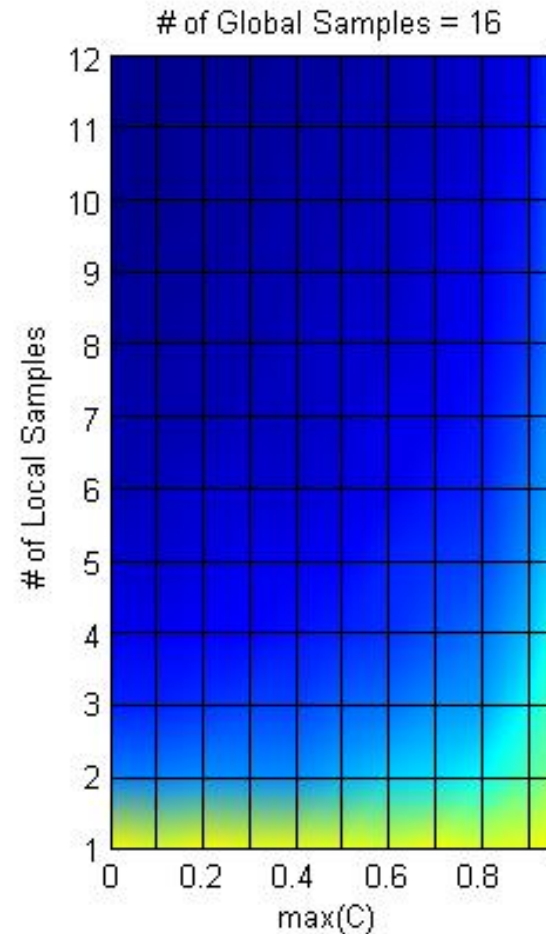
# Error B

- **Low Density (4x4 grid of 16 samples)**
  - Reduction from 8 dB to 5.5 dB with 3 local samples
- **High Density (10x10 grid)**
  - Reduction from 8 dB to 4.5 dB with 3 local samples
- **Effect of Correlation**
  - Non-linear, but manageable
  - Up to .5, similar results

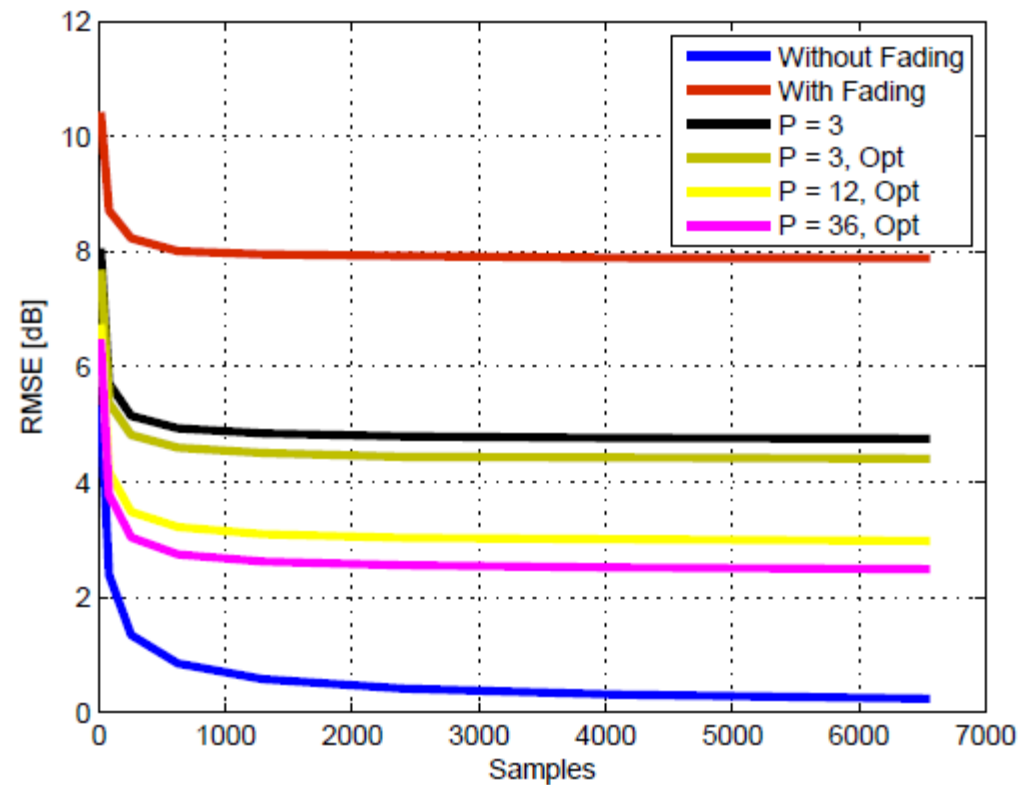


# Fading Error

- **Low Density (4x4 grid of 16 samples)**
  - Reduction from 4 dB to 3 dB with 3 local samples
- **High Density (10x10 grid)**
  - Reduction from 6 dB to 2.5 dB with 3 local samples
- **Fading Error**
  - Reduced, but not completely



# Mitigation of Aliasing by Local Averaging







# Conclusions

- Fading contribution to DRFM maps increases estimation error.
- Estimating DRFM maps using global points is not enough.
- Local power averaging reduce fading contribution and increase estimation accuracy.
- Correlation between local sampling points increase estimation error.

# References

- [1] D. Denkovski, V. Atanasovski, L. Gavrilovska, J. Riihijarvi, and P. Mahonen, "Reliability of a radio environment map: Case of spatial interpolation techniques," in *7th International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, 2012, pp. 248–253.
- [2] G. Mateos, J.-A. Bazerque, and G. Giannakis, "Spline-based spectrum cartography for cognitive radios," in *Conference Record of the Forty-Third Asilomar Conference on Signals, Systems and Computers*, 2009, pp. 1025–1029.
- [3] C. Phillips, M. Ton, D. Sicker, and D. Grunwald, "Practical radio environment mapping with geostatistics," in *IEEE International Symposium on Dynamic Spectrum Access Networks (DYSPAN)*. IEEE, 2012, pp. 422–433.